The Scaling of Seismic Energy with Moment: Simple Models Compared with Observations

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Chapman Conference on Radiated Energy and the Physics of Earthquake Faulting
Portland, Maine
June 14, 2005

UCRL-PRES-213111

This work was performed under the auspices of the U.S. Department of Energy by University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

How Do Earthquakes Scale?

Do the earthquake source properties of scaled energy, rupture velocity, and stress drop increase with magnitude?

Small event(s) maybe one patch on a larger fault with moment M_1, energy E_1 scaled energy $\tilde{e}_1 \sim E_1/M_1$, rupture velocity V_1 and stress drop □□$_1$

Large event (e.g. future magnitude 8 quake) is all patches with total moment M_N, energy E_N, scaled energy $\tilde{e}_N \sim E_N/M_N$, rupture velocity V_N and stress drop □□$_N$

What is the relationship between small and large parameters? For example does $\tilde{e}_1 = \tilde{e}_N$ or is $\tilde{e}_N > \tilde{e}_1$? Similar questions apply to all parameters.

There is currently a lack of consensus within the geophysical community on this issue despite many recent studies

Does earthquake scaled energy increase with magnitude?

Yes	No
Kanamori et al. (1993)	
Abercrombie (1995)	Choy and Boatwright (1995)
Mayeda and Walter (1996)	McGarr (1999)
Izutani and Kanamori (2001)	Ide and Beroza (2001)
Prejean and Ellsworth (2001)	Ide et al. (2003)
Richardson and Jordan (2002)	Yamada et al (2004)
Mori et al. (2003)	Matsuzawa et al. (2004)
Stork and Ito (2004)	Imanishi et al. (2004)
Mayeda et al (2005)	Prieto et al. (2004)

- With the advent of digital broadband seismology in the early 1990's:
  - moment quantitatively estimated within an uncertainty factor of 2
  - energy quantitatively estimated within an uncertainty factor of 10

- Uncertainties in energy calculations due to corrections for attenuation, radiation inhomogeneities and bandwidth.

Scaled Energy (Apparent Stress) Observables

Static

"How much did the fault move?"

Quantitative measure is seismic Moment

M_o= □ (avg. slip) (fault area)

(local, regional and teleseismic methods generally agree to within a factor of 2)

Dynamic

"How fast did the fault move?"

Quantitative measure is radiated seismic Energy

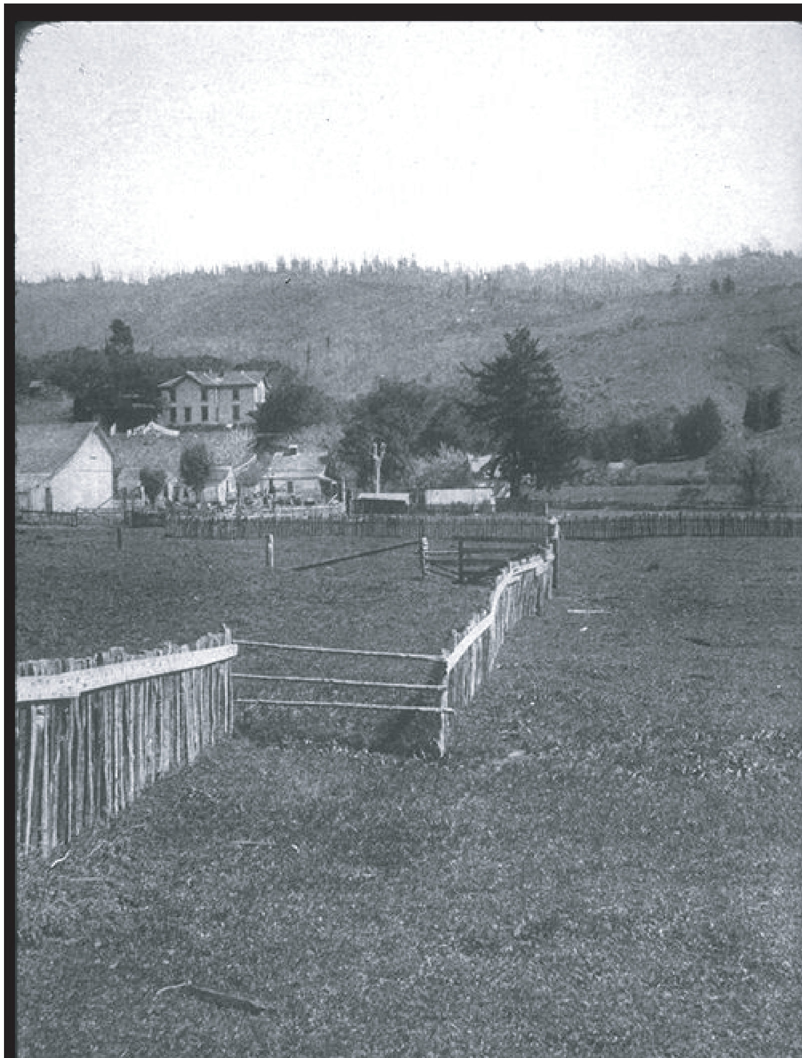
E_s = □ (avg. slip) (fault area)

(different methods often disagree by factors of 2-10)

Scaled Energy ẽ ~ Dynamic/Static

$\tilde{e} = E_s/M_o$

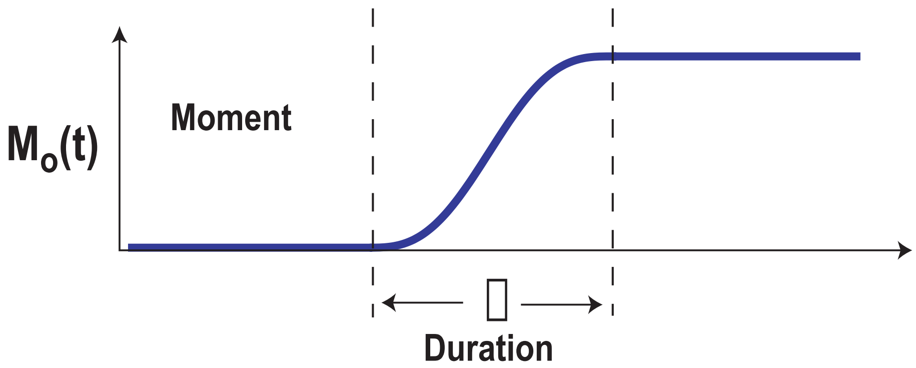
(Apparent stress =□ẽ)



1906 S. F. Earthquake
2.5 m offset near Bolinas
(photo by C.K. Gilbert)

Simple Seismic Models

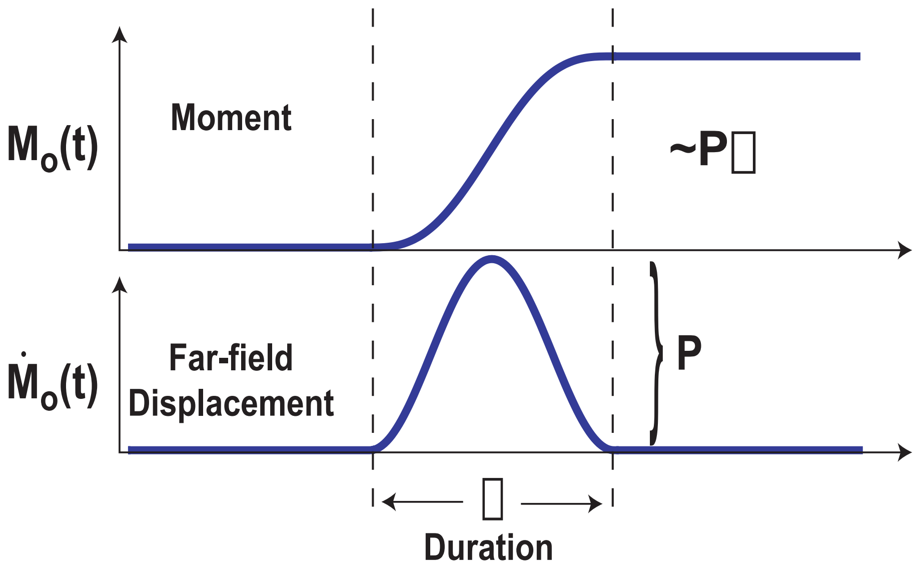
Time Domain



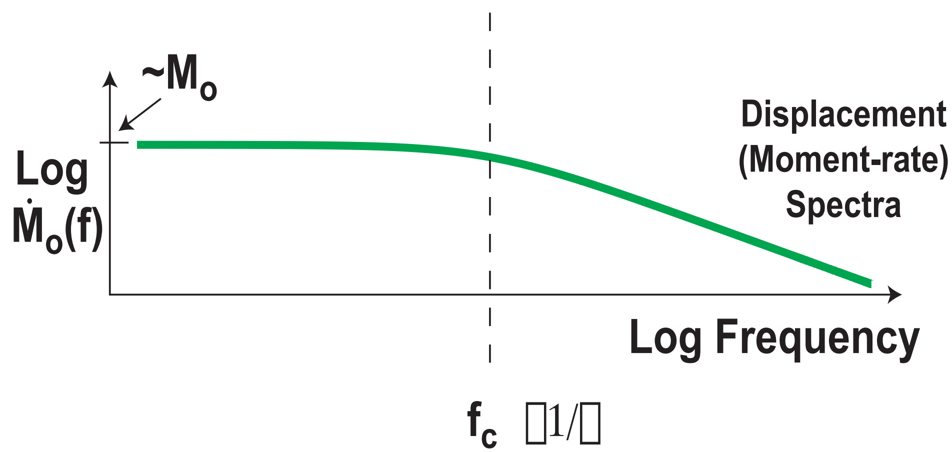
The shape of the moment time function shown here is arbitrary

Simple Seismic Models

Time Domain



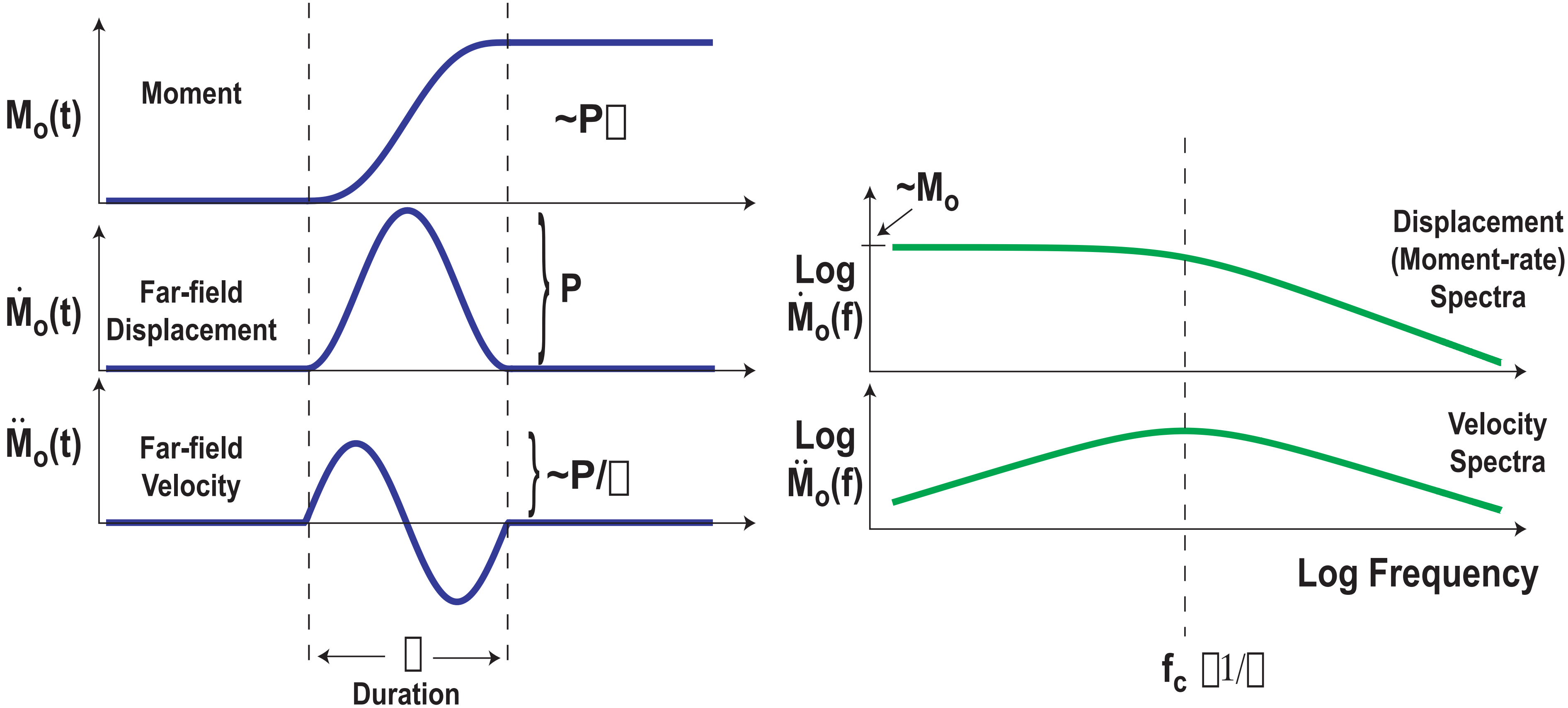
Frequency Domain



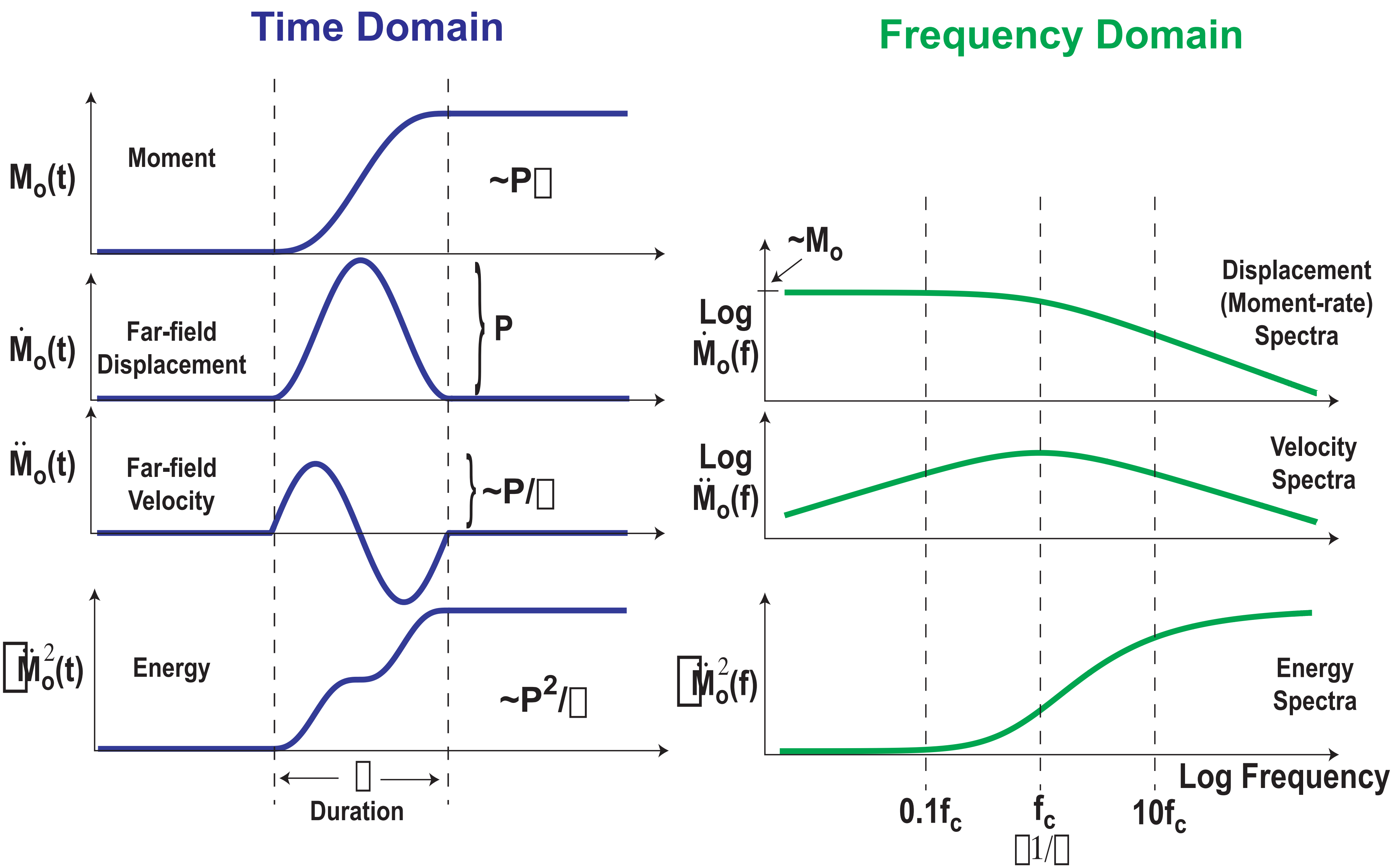
Simple Seismic Models

Time Domain

Frequency Domain

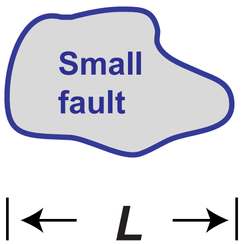


Simple Seismic Models

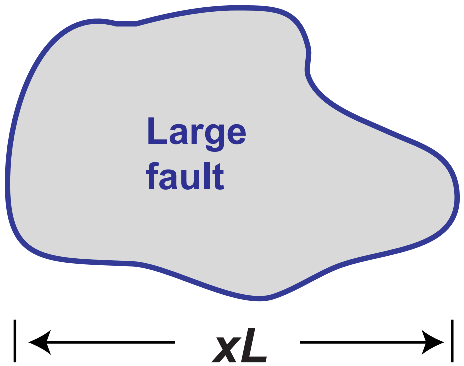


Self-Similar Earthquake Scaling

Quake



Scaled Quake



Area = A ~ L^2

Slide = $\bar{u}$ ~ $\frac{□□}{□}$L

M_o = □$\bar{u}$A ~ □□L^3

Duration = □ ~$\bar{u}$/V ~L

Energy = E ~ P^2/□

Area = x^2A

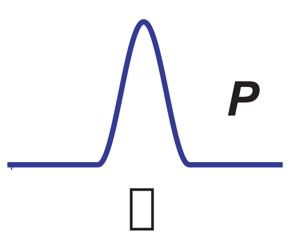
Slip = x$\bar{u}$

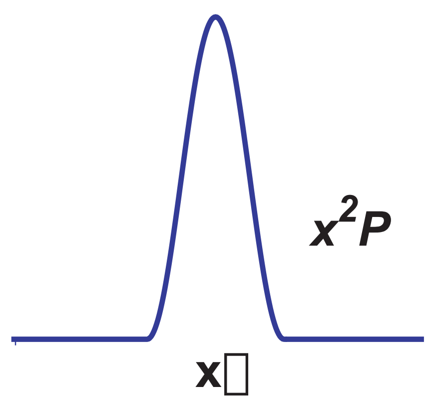
M_o = $x^3 M_o$

Duration = x□

Energy = x^3E

$\dot{M}_o(t)$

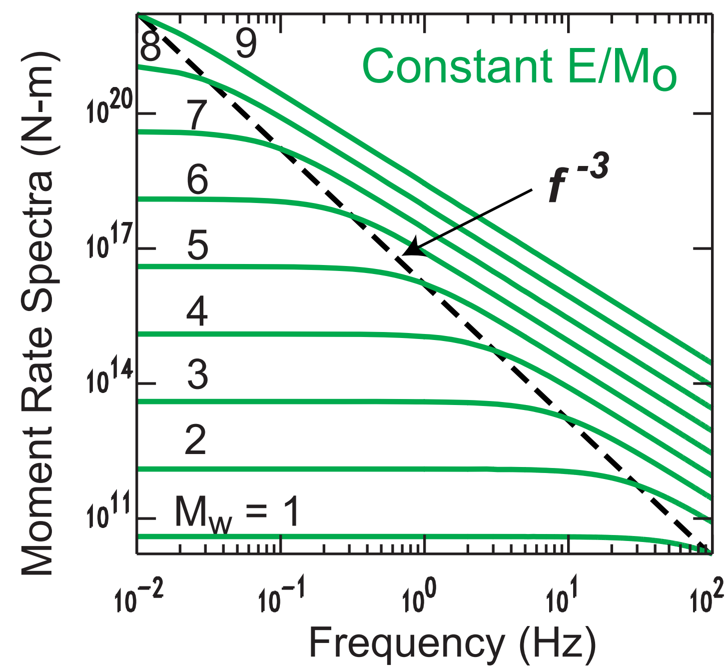




Implicit □□ and V are constant

M_o ~ □□L^3 ~ □□V^3□^{3} ~ (□□V^3)f_c^{-3}

Product □□V^3 is constant



Aki (1967) and Brune (1970) models, any model invariant under f^{-3} scaling

(see also Prieto et al 2004; Kanamori and Rivera, 2004)

Non-self-similar earthquake scaling: Case of same spectral shape but different corner frequency behavior

1) Changes in □□ and/or V with size

(e.g. large have higher □□ and/or V than small)

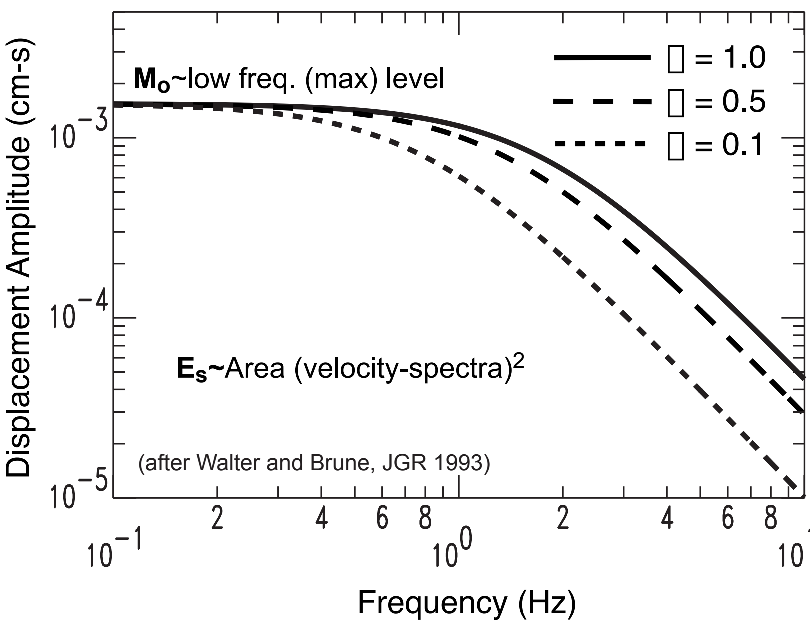
2) Changes in efficiency with size

(large more efficient than small)

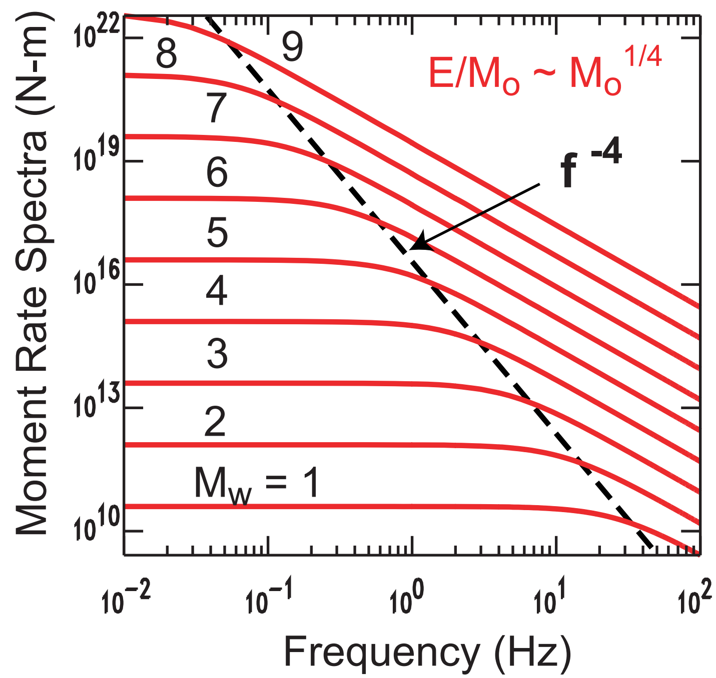
Kanamori and Rivera (2004):

- □ $M_o \sim f_c^{-(3+□)}$
- □ $\tilde{e} \sim M_o^{□/(3+□)} \sim (□□V^3)$

Constant moment shear crack spectra for different seismic efficiencies

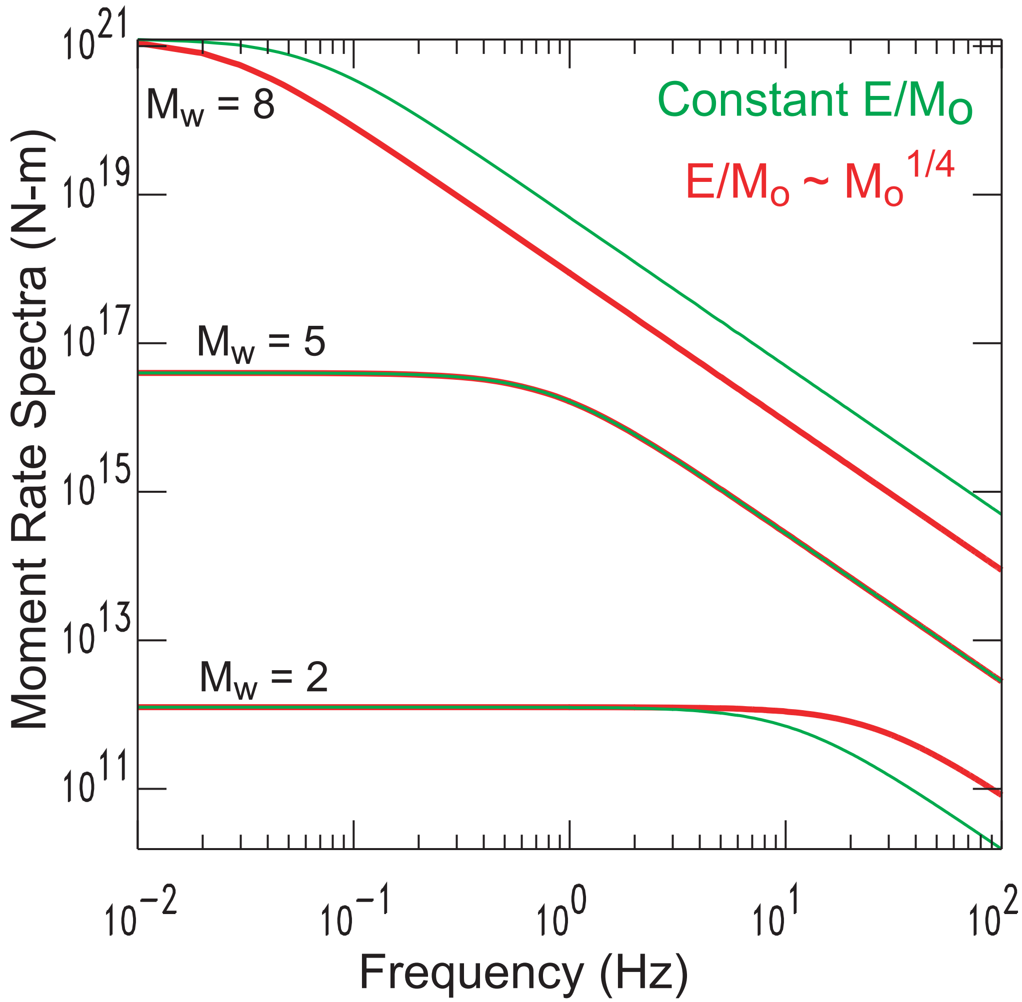


An Example where $M_o \sim f_c^{-4}$



Spectral shape invariant as f^{-4}

The differences between the spectral scaling models are clearest for very small and very large events



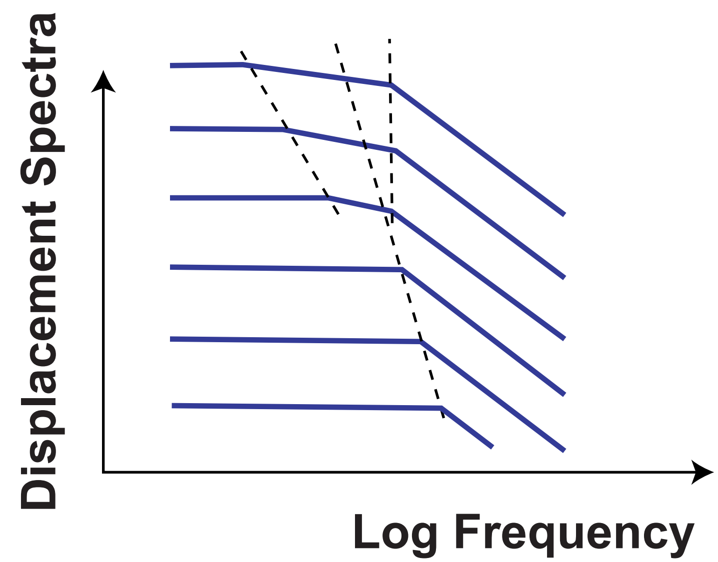
Large events are infrequent, small events need high frequencies

Non-self-similar earthquake scaling: Case of spectral shape change with size

1) Changes in fault shape with size
(e.g. equidimensional to unidimensional growth)

2) Changes in fault physics with size
(e.g. dynamic friction changes with size)

Example: Intermediate falloff



Energy Measurement Issues

1) Energy is broadband -

- a) want at least 2 orders of magnitude bandwidth in frequency
- b) high and low frequency corrections can still be important

2) Path and site effect corrections - can have a large effect

3) Majority of energy is radiated from the source as S-waves
(e.g. Boatwright and Fletcher, 1984):

$$q = \frac{3}{2}\left(\frac{\alpha}{\beta}\right)^{5}\left(\frac{f_{cS}}{f_{cP}}\right)^{3}$$

q varies from 4.5 to 23.4 for a Poisson solid as P to S corner frequency ratio varies from 1.73 to 1.

Want to focus on S-wave for energy measure or make big corrections.

4) Source directivity and inhomogeneities - want to average over azimuth and takeoff angle

An example of looking at E/Mo scaling by correcting and combining multiple studies

The paper by Ide and Beroza (GRL, Sept. 2001) corrects some small event studies and puts multiple studies on the same plot

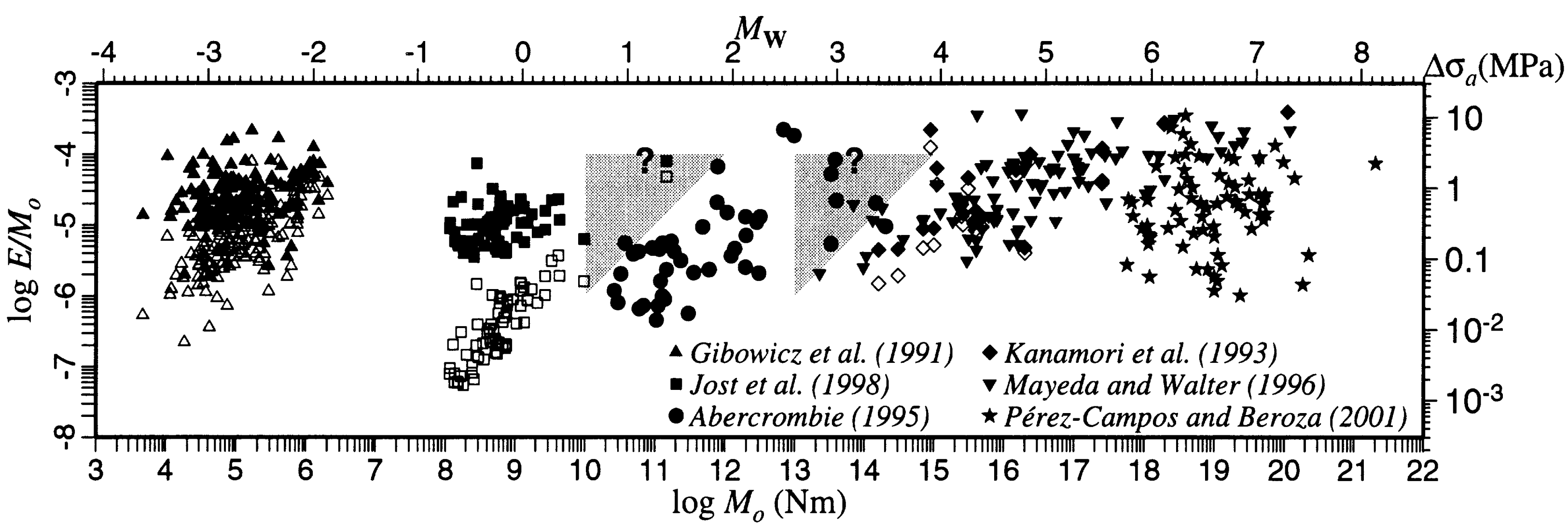


Figure 3. Energy-seismic moment ratio and apparent stress. Each symbol denotes a different data set as shown. Solid symbols are adjusted values or those that already accounted for the missing energy. Open symbols are the original values. Two gray triangles are the area where possibly missing events would locate.

Geophysical Research Letters, volume 28, pages 3349-3352, September 2001

After correcting some of the studies for limited bandwidth they find apparent stress appears constant, though with a large variance

A question remains as to whether combining these disparate studies makes an "apples" to "oranges" comparison

The paper by Ide and Beroza (GRL, Sept. 2001) with my annotations in red

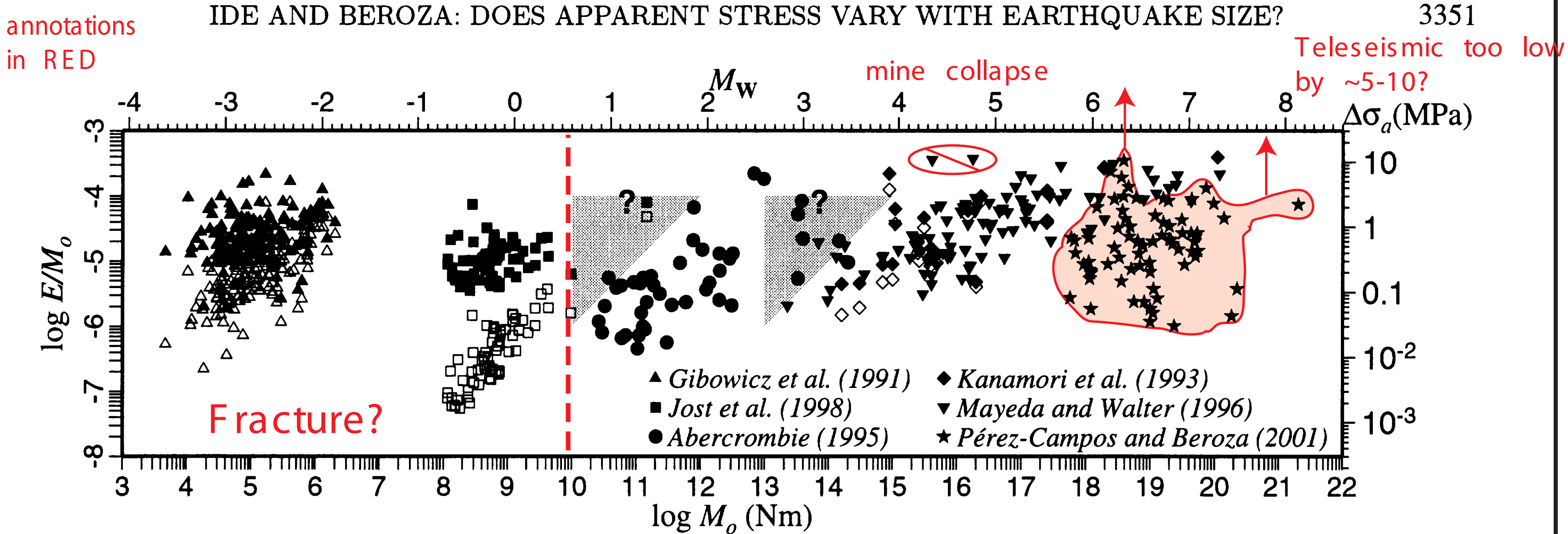


Figure 3. Energy-seismic moment ratio and apparent stress. Each symbol denotes a different data set as shown. Solid symbols are adjusted values or those that already accounted for the missing energy. Open symbols are the original values. Two gray triangles are the area where possibly missing events would locate.

Geophysical Research Letters, volume 28, pages 3349-3352, September 2001

After making changes indicated in red, the apparent stress appears to increase with seismic moment, again with large variance

Borehole studies attempt to minimize site and path effects allowing scaling studies of very small events

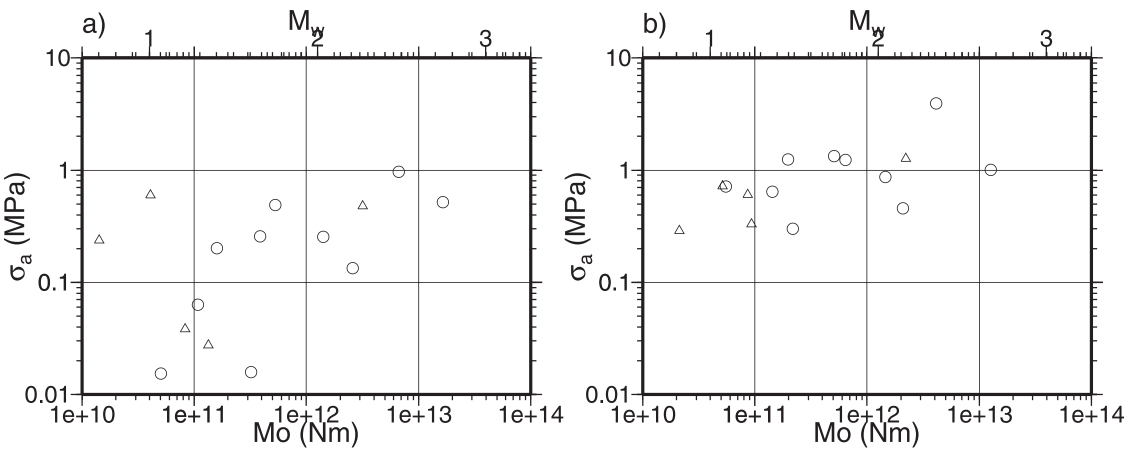


Figure 9. Comparison between seismic moment and apparent stress in constant Q analysis and spectral ratio analysis. Circle and triangle represent 1997ñ1998 events (C1) and 1992 events (C2), respectively.

Ide, Beroza, Prejean and Ellsworth, JGR 2003

Long Valley
2054 m Borehole

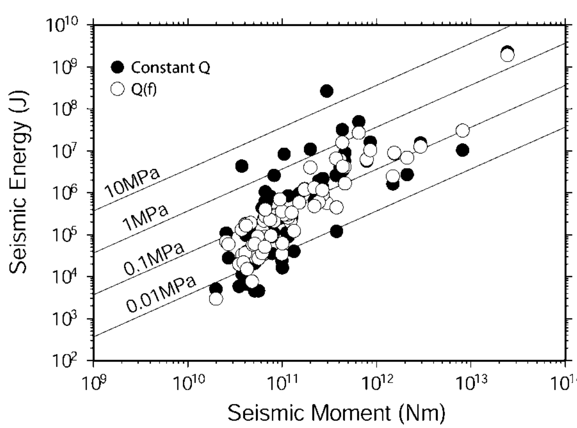


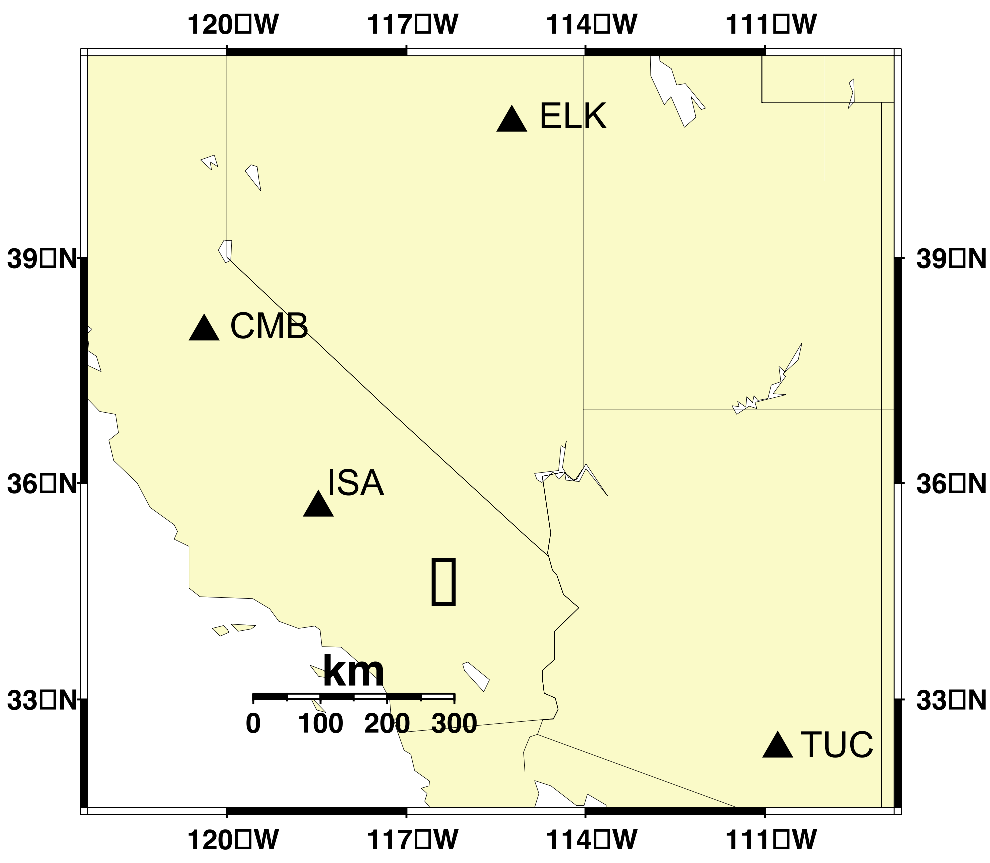
Figure 13. Scaling of $_a$ with M_0 for constant-Q and $Q(f)$ model analyses. Lines of constant $_a$ are shown.

Stork and Ito, BSSA 2004

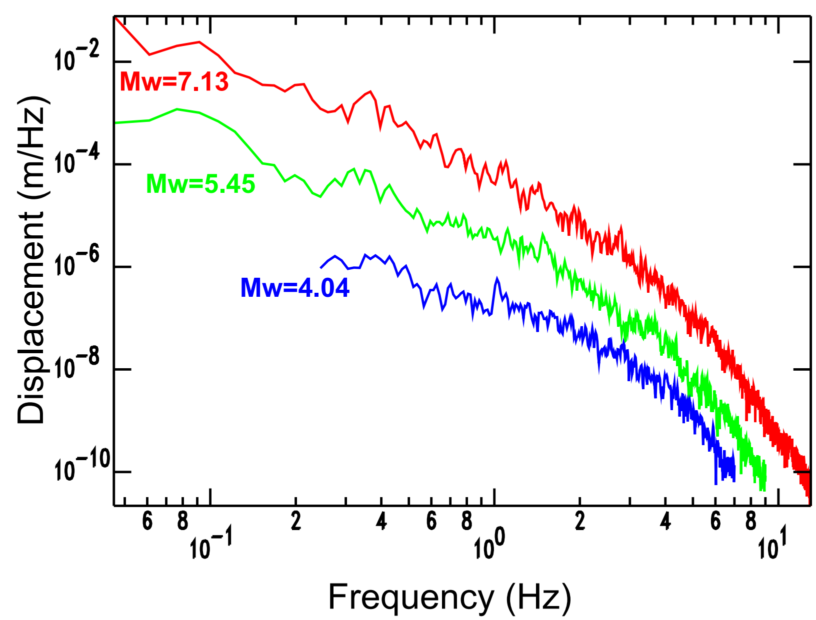
Western Nagano
800 m deep Borehole

We evaluate energy scaling within the 1999 Hector Mine earthquake sequence using constant stations and paths

At regional distance Lg is on scale for main shock with bandwidth for aftershocks Mw > 3.75

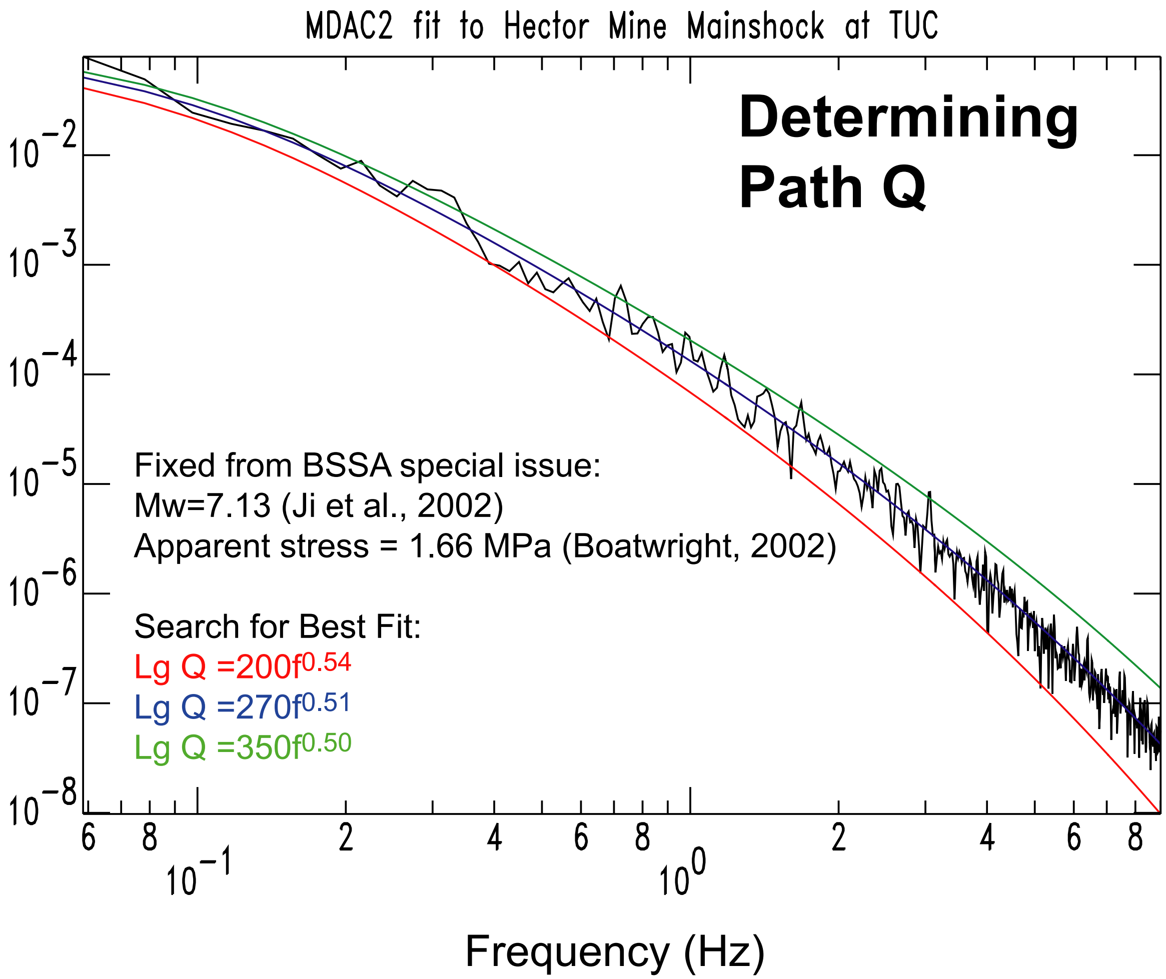


Three-component Lg spectra at ELK



Energy determined by integrating spectra corrected for site and path with low and high frequency corrections

We use independent mainshock moment and apparent stress to determine path correction

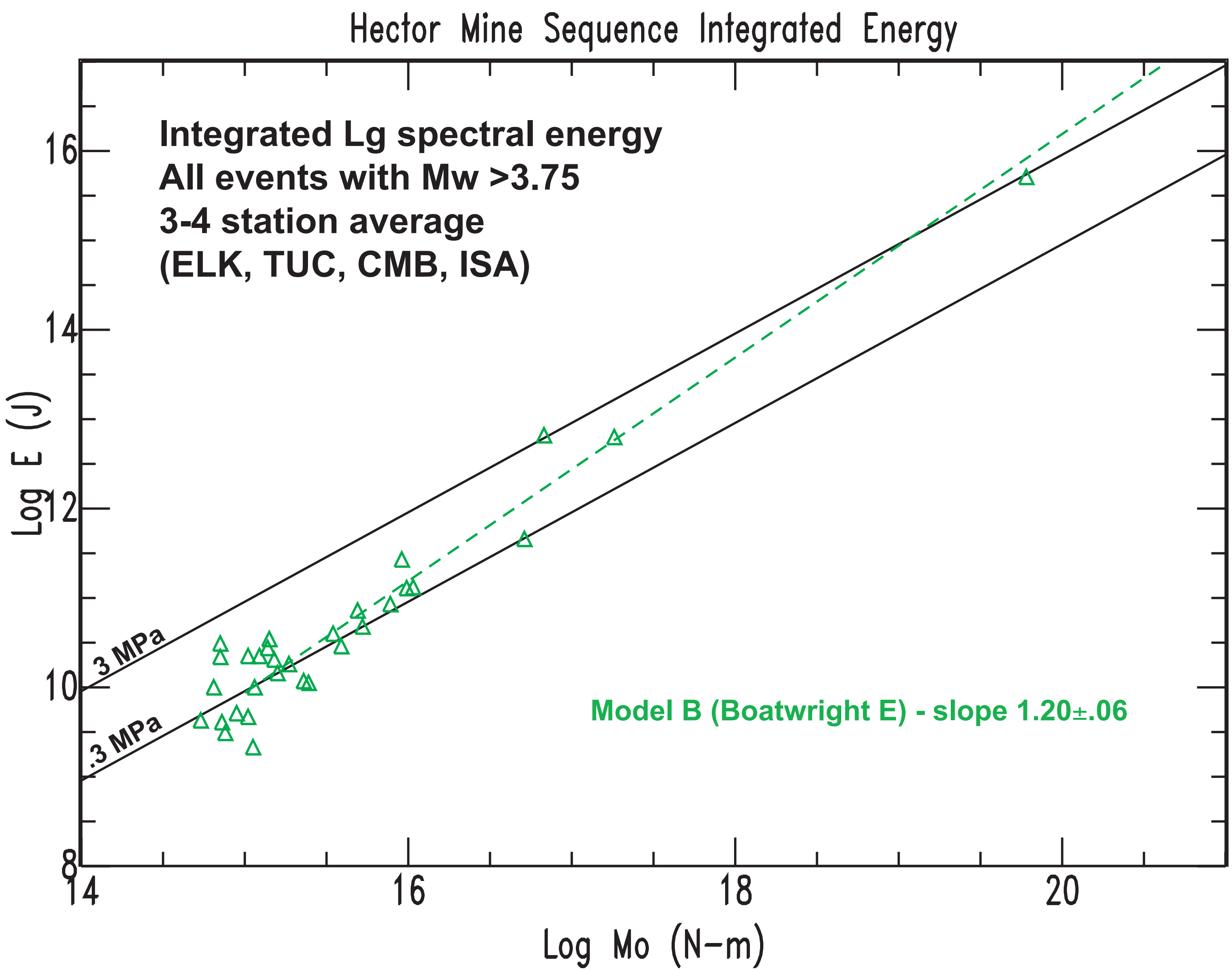


Spectra are first corrected for geometrical spreading using a Street et al. (1975) type model:

$$G(R) = \begin{cases} \dfrac{1}{R} & \text{where} \quad R < R_o \\ \dfrac{1}{R_o}\left(\dfrac{R_o}{R}\right) & \text{where} \quad R \geq R_o \end{cases}$$

All Hector Mine events at TUC are then evaluated using this correction. Similarly sepearate corrections are determined for each station.

The Hector mine data shows E/Mo scaling with size for path corrections that match main shock measures



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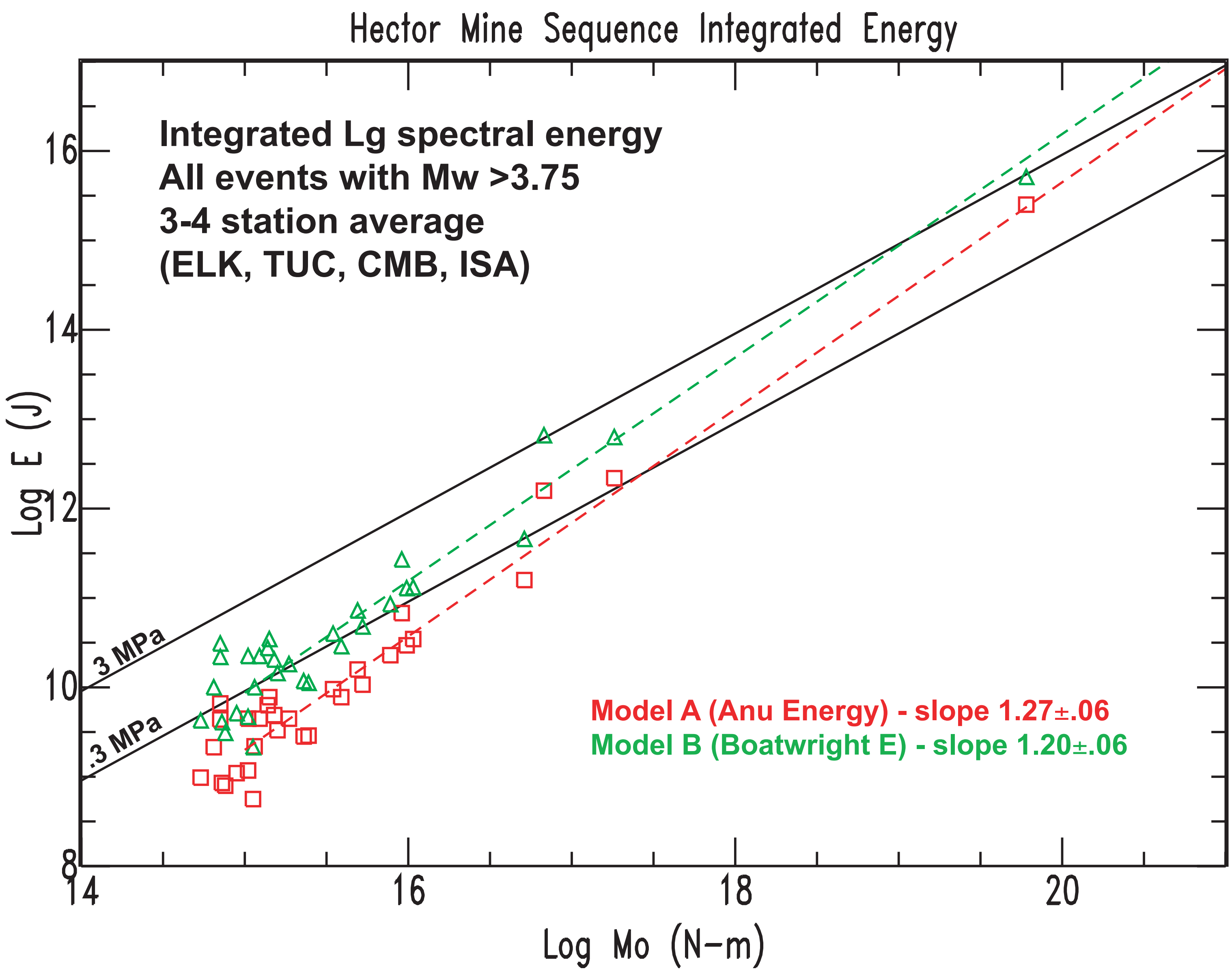

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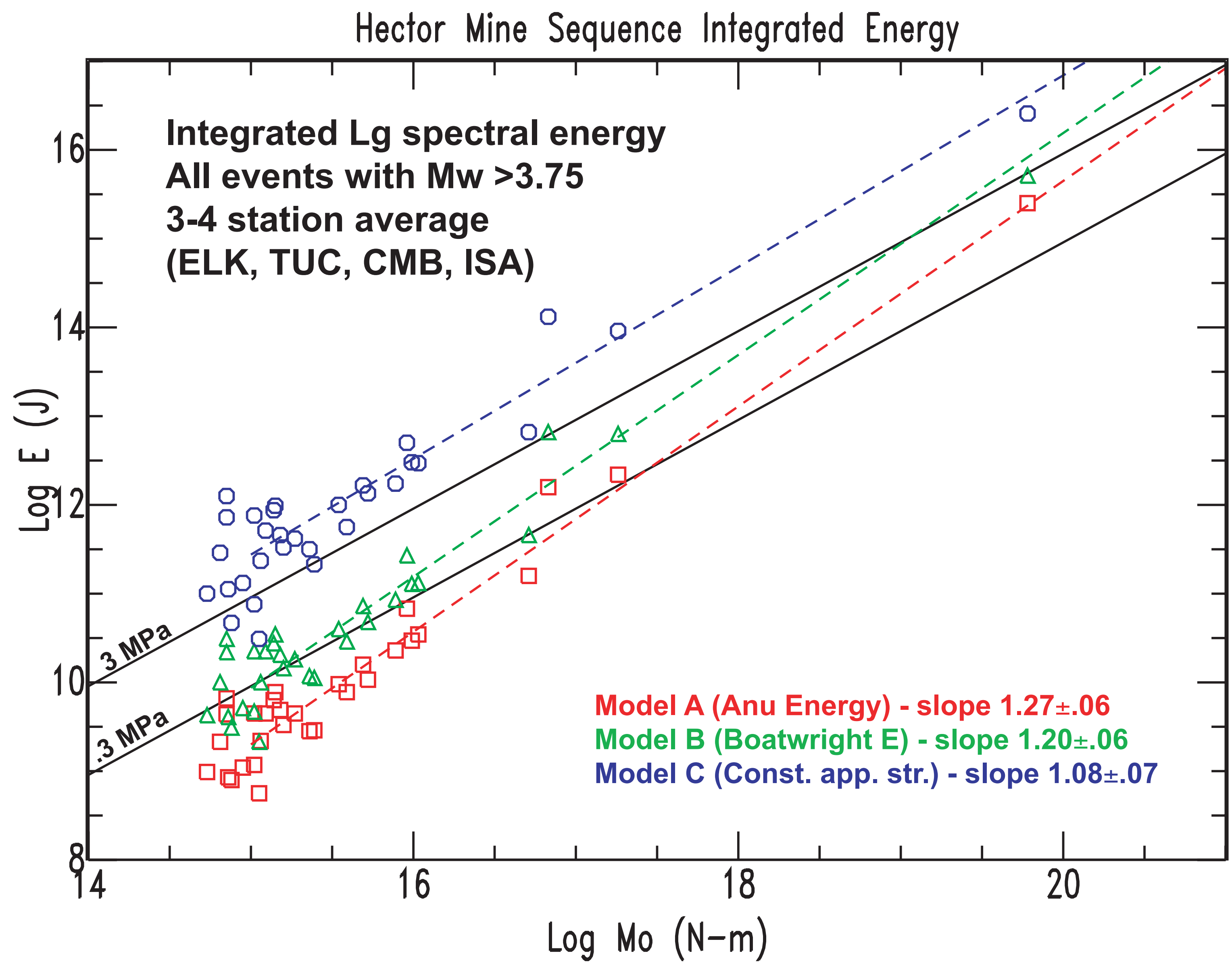
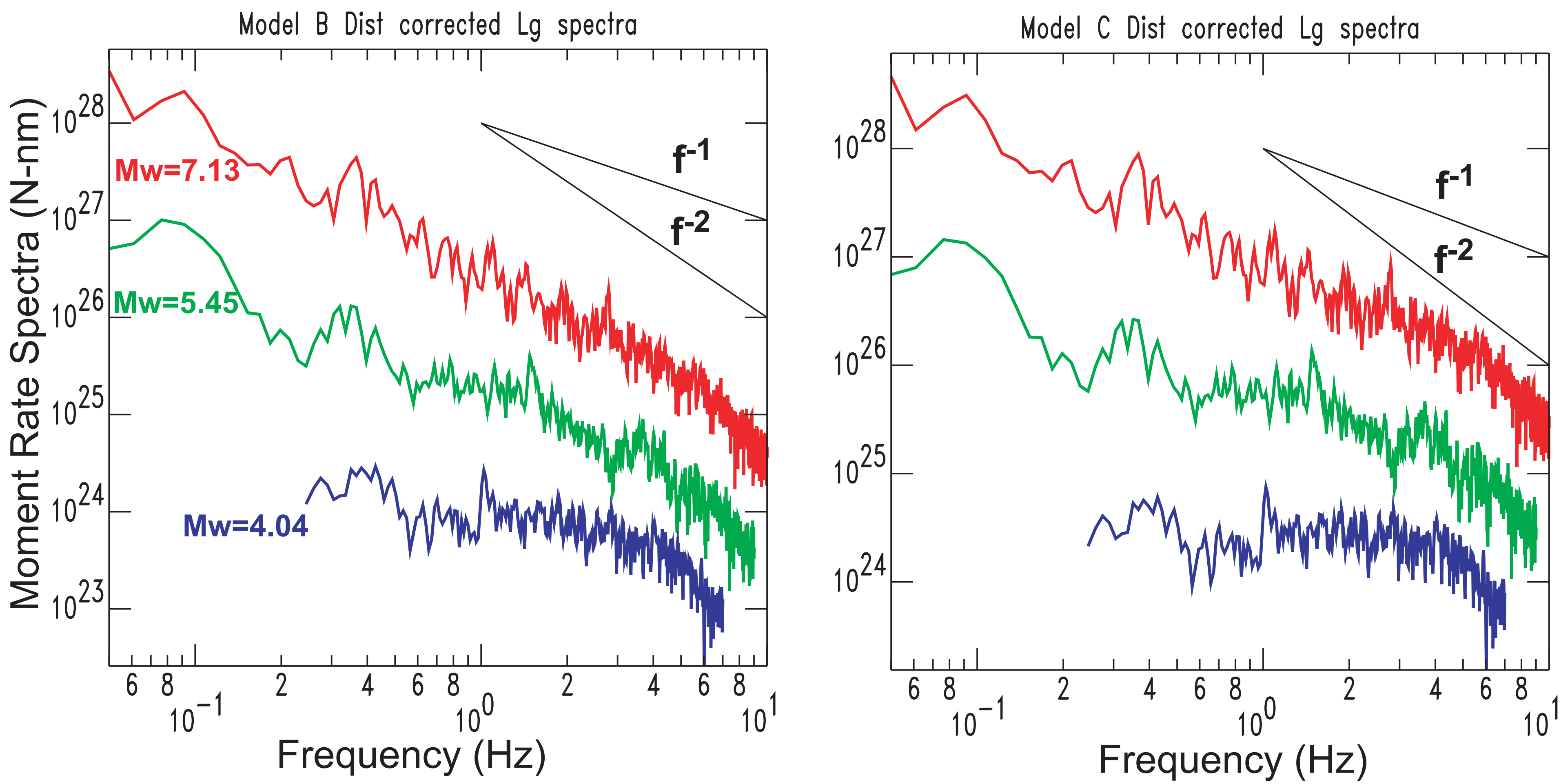
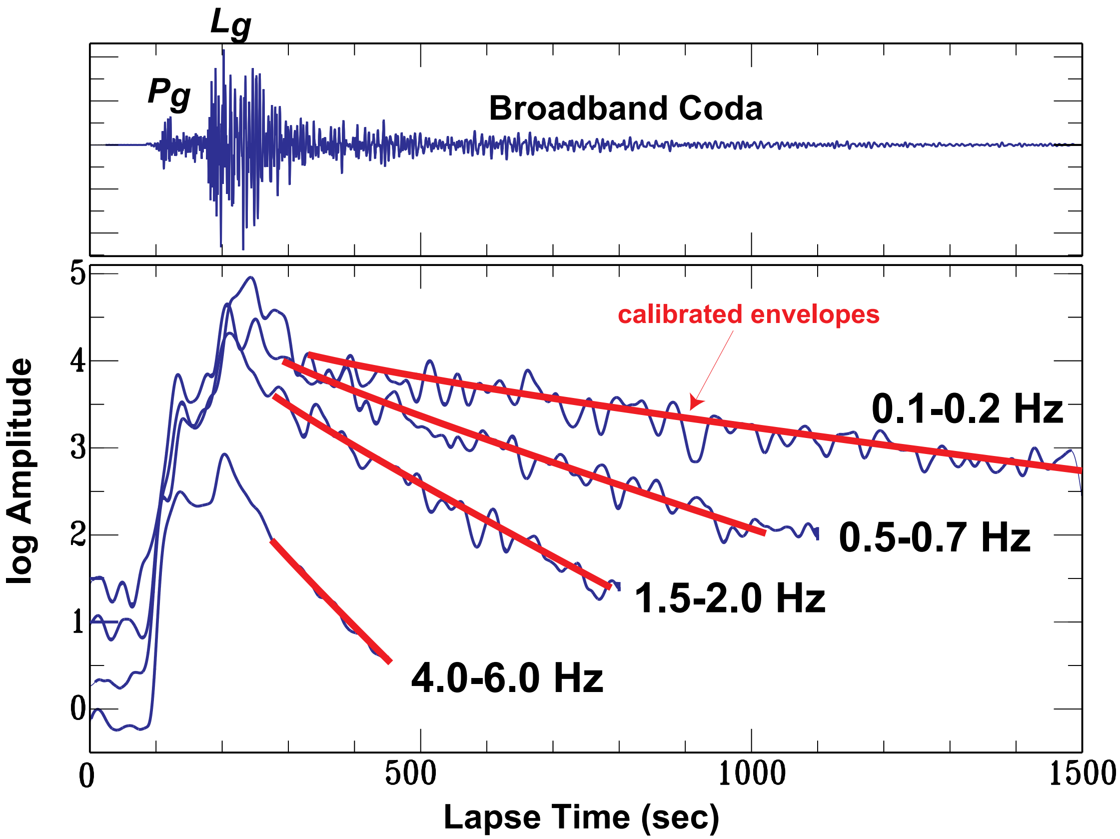


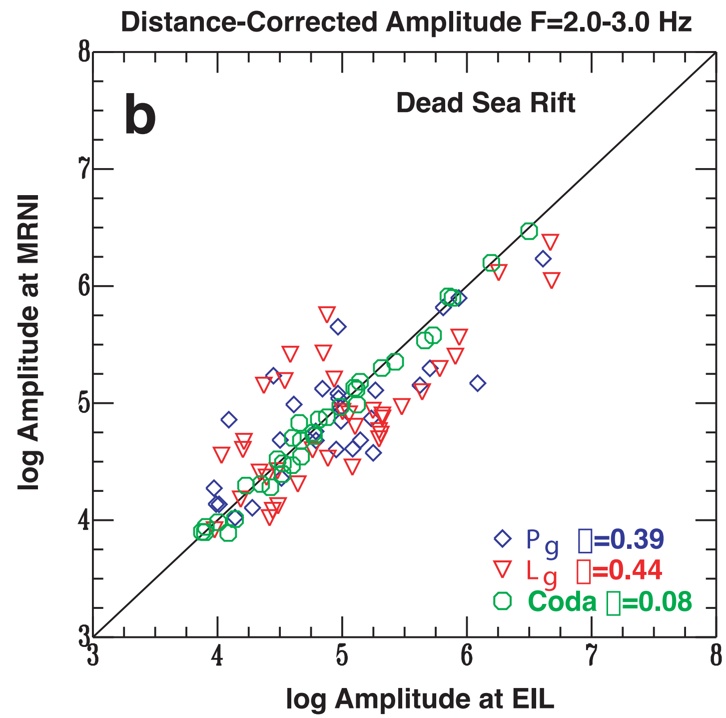
The Model C path corrected spectra do not have the same shape indicating non-similar scaling even if E/Mo is constant



For Hector Mine Lg spectra there does not seem to be a set of path corrections that can produce both constant E/Mo and similarly shaped spectra

The local to regional coda envelope technique method has a number of advantages for determining source spectra





Advantages of using coda technique include:

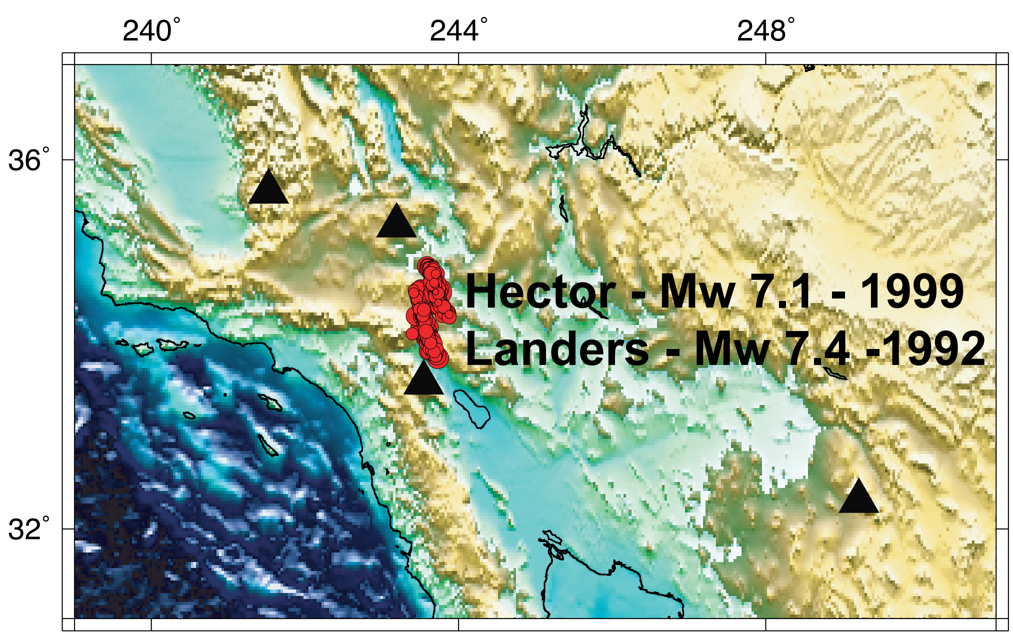
1) Averages over source giving very stable amplitudes for single station
2) Amplitudes formed into smooth spectra provide Mo and energy
3) Clipping and windowing phases are not a problem
4) Technique is completely empirical with only EGF assumption

(see Mayeda et al., 2003)

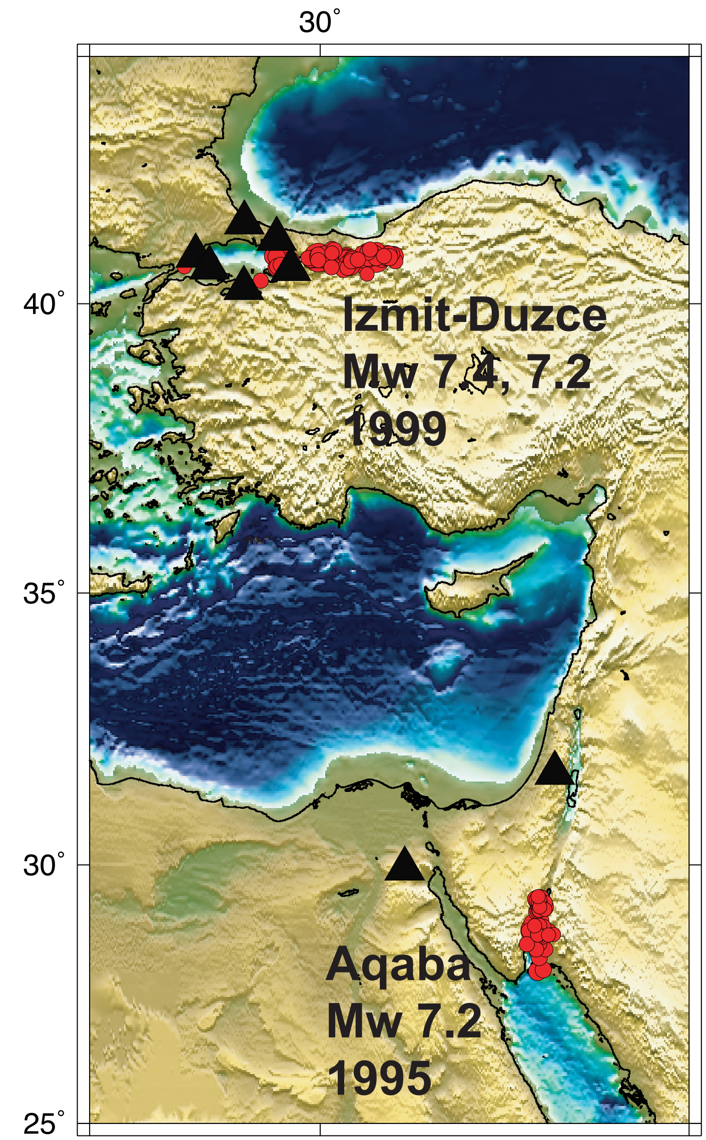
We carried out coda analysis on four large strike-slip sequences: Landers, Hector Mine, Izmit-Duzce and Aqaba

Mayeda, Gok, Walter and Hofstetter, GRL May 2005

Western U.S.

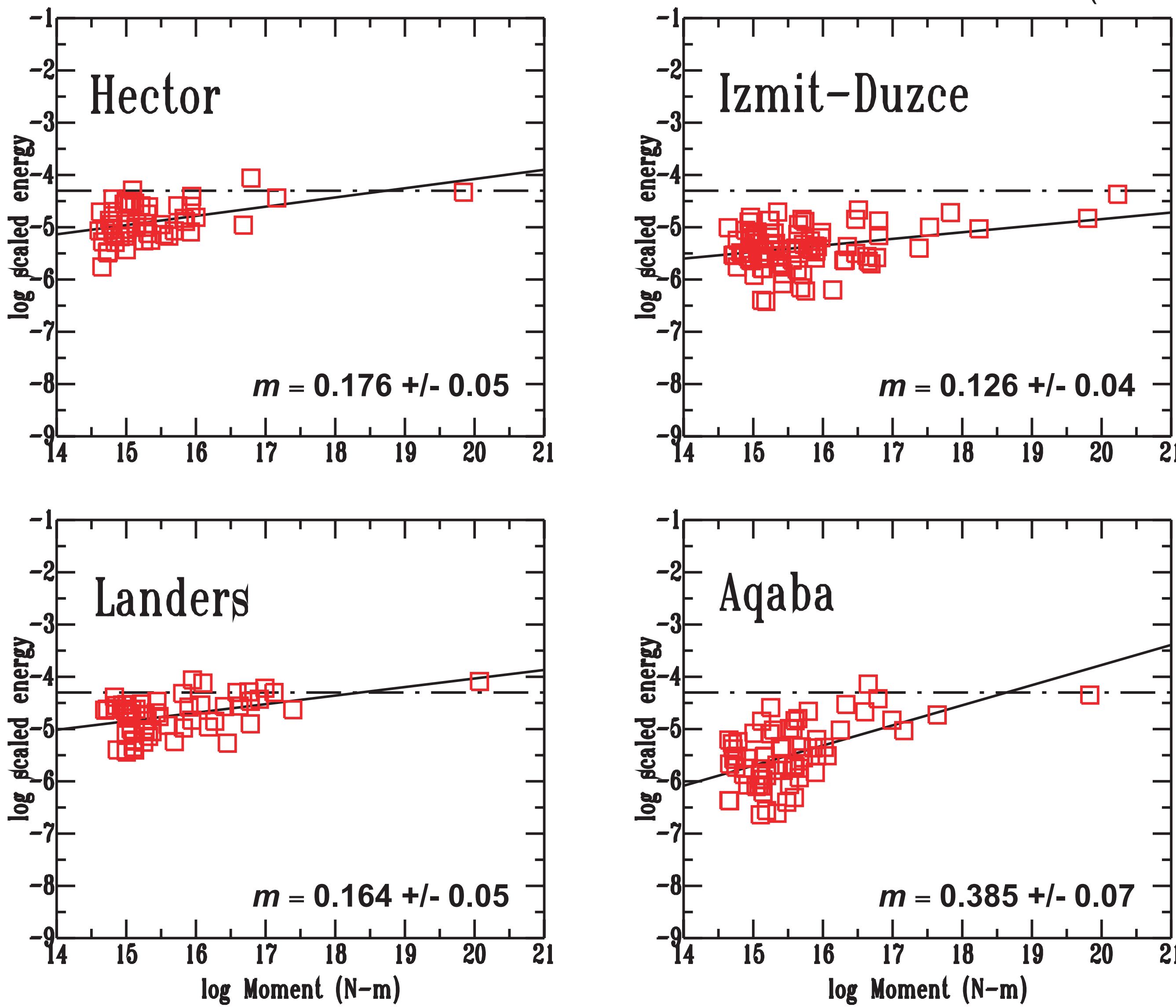


Middle East



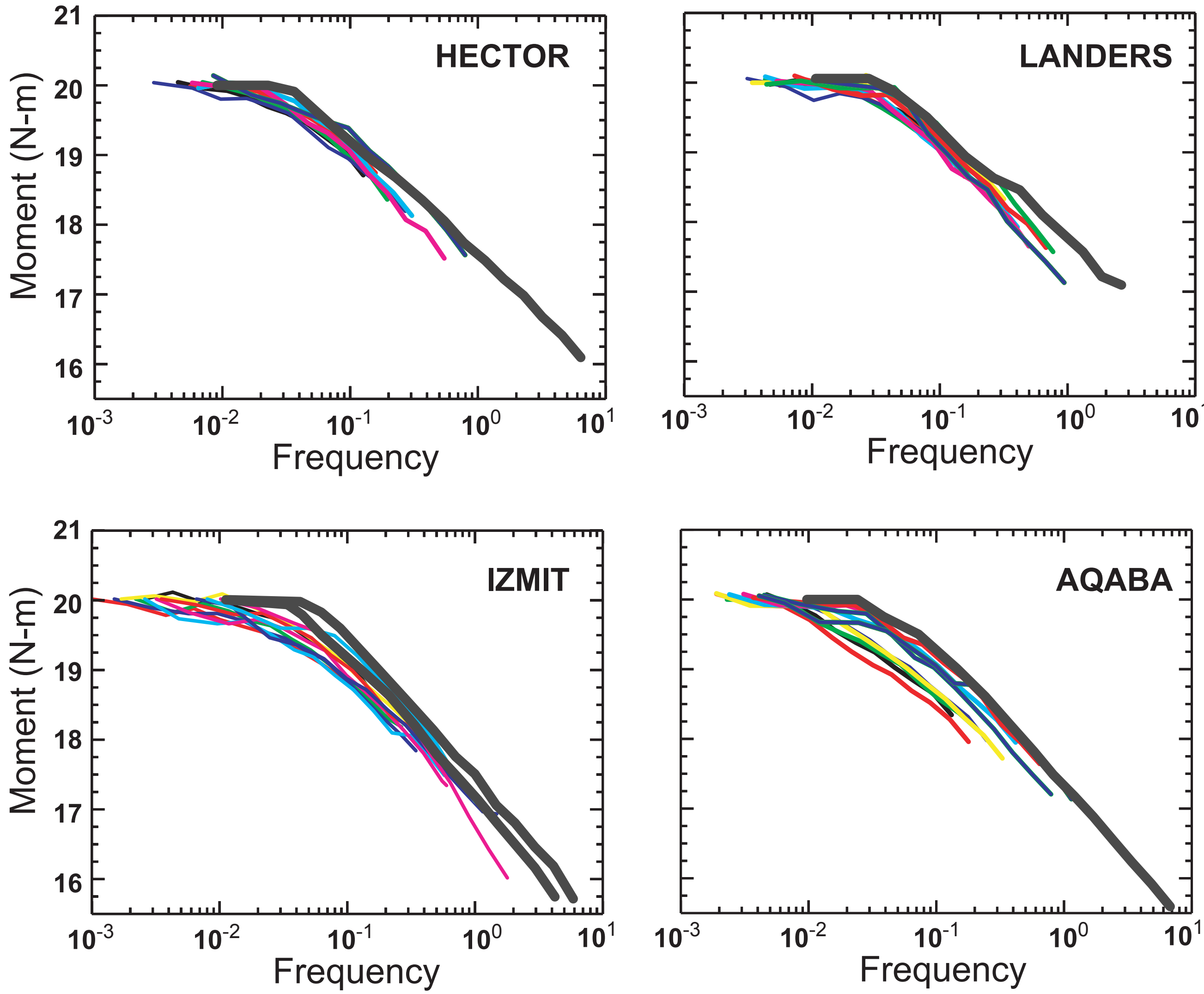
Coda results show non-constant scaled energy behavior within all sequences

(from Mayeda et al., 2005)



Aqaba has shallow normal ridge events mixed with strike-slip at small Mw

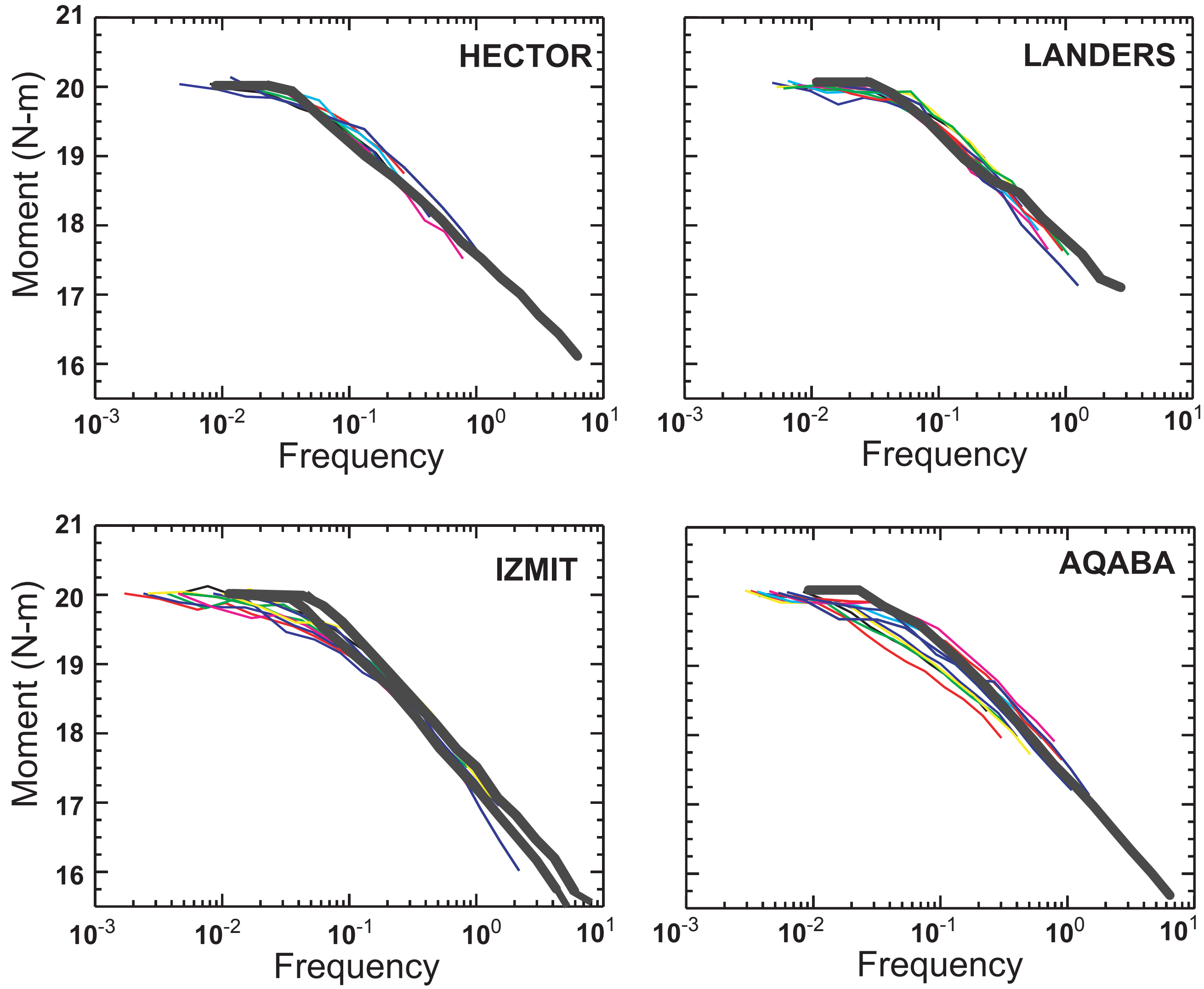
Sliding all spectra along a f^{-3} line as suggested by Prieto et al. (2004) shows spectra are not self-similar



(from Mayeda et al., 2005)

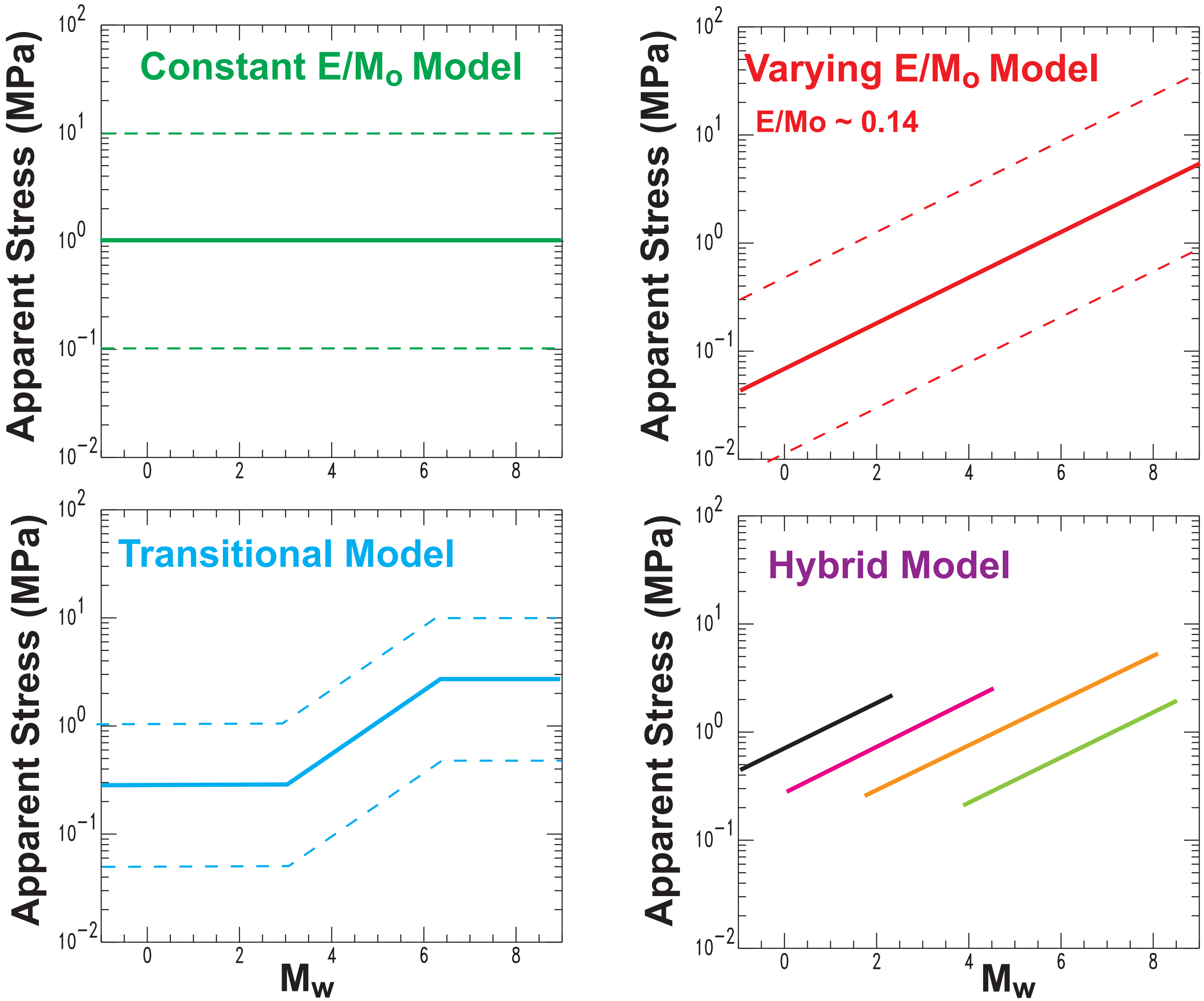
Sliding all spectra along a $f^{-3.5}$ line improves spectra match

(from Mayeda et al., 2005)



This is = 0.5 in Kanamori and Rivera (2004) notation and consistent with E/Mo ~ 0.14

In summary there appear to be many possible models consistent with particular data studies for Mw 1-7



Models are schematic.

In the Mw 1-7 range the models may be hard to differentiate.

Need single consistent technique that can span large Mw range.

Quantitative constraint of apparent stress variance with magnitude is needed!

Summary

While energy estimates and our understanding of the care needed with all the corrections applied has significantly improved, we currently remain unable to constrain the overall behavior of scaled energy with size.

Scaling effects may be subtle - we need to look across many orders of moment magnitude in a consistent way (same stations, paths, combining local and regional data, larger dynamic range, more bandwidth, etc.).

Scaled energy may vary with depth, fault age, tectonic setting, recurrence interval so we need to find better ways to compare across regions.

Differences between models are greatest at extreme sizes (large and small) and these may be the focus future studies (e.g. boreholes, great quakes)

Example Sumatra: Time function from Ishii et al, 2005.

Are aftershocks Self-similar??